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EVAPORATION OF LIQUIDS FROM CYLINDRICAL VESSELS UNDER CONDITIONS OF FREE CONCENTRATIONAL CONVECTION IN A GAS PHASE

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An analytical solution is obtained for the axisymmetric problem of free concentrational convection in a vapor-gas mixture with isothermal evaporation of liquids from open cylindrical vessels. Formulas are derived to calculate concentration fields, local and integral mass fluxes of vapor. A comparative analysis of the results of analytical and numerical simulation is carried out for the processes of the evaporation of liquids under the conditions of convective mass transfer.

During evaporation of liquids whose molar mass is smaller than that of the gas in whose medium the evaporation occurs, conditions can develop under which the hydrodynamic stability of the gas phase is disturbed. In this case, mass transfer of the vapor takes place in the free convection regime and is described by the system of equations [1]:

$$-\nabla \mathrm{p}_m + \eta \Delta \mathrm{V} - \rho \mathrm{g} = 0\,, \tag{1}$$

$$\mathrm{v}\cdot\nabla\rho_1 - D\Delta\rho_1 = 0\,, \tag{2}$$

$$\mathrm{div}\,\mathrm{v} = 0\,, \tag{3}$$

where p_m, ρ, v are the pressure, density, and velocity of the gas-vapor mixture; η, D are the coefficients of the dynamic viscosity and diffusion; $\rho_1 = m_1/V$ is the concentration of the vapor in the mixture.

The steady-state evaporation regime in cylindrical vessels can be analyzed in the approximation $v_r \ll v_z$ (where v_r and v_z are the radial and axial velocity components), which is the more correct the smaller the radius of the vessel R in comparison with its height H. In this case we do not take into consideration the processes occurring in the adjoining layer of the vapor-gas mixture, whose thickness is negligibly small as compared with the dimensions of the vessel. For an axisymmetric variant of convection, which was studied by numerical methods in [2], in this approximation we can obtain an analytical solution to the system of Eqs. (1)-(3) which in cylindrical coordinates takes the form:

$$-\frac{\partial p}{\partial z} + \eta\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right) - \rho g = 0\,, \tag{4}$$

$$v_z\frac{\partial\rho_1}{\partial z} - D\left(\frac{\partial^2\rho_1}{\partial r^2} + \frac{1}{r}\frac{\partial\rho_1}{\partial r} + \frac{\partial^2\rho_1}{\partial z^2}\right) = 0\,. \tag{5}$$

The pressure and density of the mixture will be written as

$$p_m = -\rho_0 g z + p\,, \quad \rho = \rho_0 + \rho'\,, \tag{6}$$

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where ρ_0 is the density of the mixture in the medium surrounding the vessel; p, ρ' are the excess values of the pressure and density caused by the concentrational inhomogeneity of the mixture.

Assuming the change in the pressure to be much smaller than that in the molar mass of the mixture M, we write $\rho' = \rho(1 - \rho_0/\rho) = (1 - M_0/M)$, just as in [2]. Using the expression for the molar mass of the mixture $M = M_1M_2/(\omega_1M_2 + \omega_2M_1)$, where $\omega_1 = \rho_1/\rho$, $\omega_2 = \rho_2/\rho$, M_1 and M_2 are the mass fractions and the molar masses of the components, we find

$$\rho' = \rho\frac{(M_2 - M_1)(\omega_{01} - \omega_1)}{M_1 + \omega_{01}(M_2 - M_1)}. \tag{7}$$

Substituting expressions (6)-(7) into Eqs. (4)-(5) and introducing the new concentrational variable $\psi = (\omega_1 - \omega_{01})/(\omega_{1e} - \omega_{01})$ (ω_{1e} is the equilibrium mass fraction of the vapor), we obtain a system of equations in dimensionless cylindrical coordinates $x = r/R$ and $y = z/R$:

$$-\frac{\partial F}{\partial y} + \frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial u}{\partial x} + \mathrm{Ra}\,\psi = 0, \tag{8}$$

$$u\frac{\partial \psi}{\partial y} - \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{x}\frac{\partial \psi}{\partial x} + \frac{\partial^2 \psi}{\partial y^2} = 0, \tag{9}$$

where F and u are the dimensionless pressure and velocity; Ra is the Rayleigh number defined by the expressions

$$F = \frac{pR^2}{\rho\nu D}; \quad u = v_z\frac{R}{D}; \quad \mathrm{Ra} = \mathrm{Pr}\,\mathrm{Gr}; \quad \mathrm{Pr} = \nu/D; \quad \mathrm{Gr} = \frac{\alpha g R^3}{\nu^2}, \tag{10}$$

here $\alpha = (M_2 - M_1)(\omega_{1e} - \omega_{01})/\ [M_1 + \omega_{01}(M_2 - M_1)]$, Pr and Gr are the Prandtl and Grashof numbers.

Now we will seek the function ψ in the form:

$$\psi(x, y) = \varphi(x) - Ay + B, \tag{11}$$

which allows us to transform system of Eqs. (8)-(9) in the following manner:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial u}{\partial x} + \mathrm{Ra}\,\varphi = 0, \tag{12}$$

$$\frac{\partial F}{\partial y} + \mathrm{Ra}\,(Ay - B) = 0, \tag{13}$$

$$-Au = \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{x}\frac{\partial \varphi}{\partial x}. \tag{14}$$

Applying the Δ_x operation to Eq. (12) and using Eq. (14), we obtain a biharmonic equation for the velocity u:

$$\Delta^2 u - \mathrm{Ra}\,Au = 0. \tag{15}$$

The solution satisfying the requirement for the existence of an adherent layer at the side surface of the vessel and of the continuity equation, has the form:

$$u(x) = C\left[\frac{J_0(kx)}{J_0(k)} - \frac{I_0(kx)}{I_0(k)}\right], \quad k = \sqrt[4]{\mathrm{Ra}\,A}. \tag{16}$$

After the substitution into Eq. (14), this equation makes it possible to find the function $\varphi(x)$:

$$\varphi(x) = \frac{AC}{k^2}\left[\frac{J_0(kx)}{J_0(k)} + \frac{I_0(kx)}{I_0(k)}\right]. \tag{17}$$

From the condition of impermeability of the vessel walls to vapor $\partial\varphi/\partial x|_{x=1} = 0$ we find $k = 4.6109$, and from the solution of Eq. (16) we obtain $A = k^4/\mathrm{Ra} = k^4\nu D/\alpha g R^3$, i.e., the lower limit of the range of critical values for the vapor concentration gradient $\partial\psi/\partial y$ corresponding to the instant of the disruption of the hydrodynamic stability and to the transition to the axisymmetric variant of free-convective mass transfer. It is evident that the expression, which determines the critical value of the vapor concentration gradient A, agrees in every respect with the physical meaning of the phenomenon under consideration. The transition to the convective regime of transfer will occur the more readily, the lower the viscosity of the mixture and the diffusion coefficient of the vapor and the larger the vessel radius and the difference between the molar masses of the vapor and gas.

In order to find the two unknown constants B and C, we will use the following boundary conditions: a) in a descending flow the minimum value of the vapor concentration in the outlet cross section of the vessel corresponds to the concentration in the surrounding medium, i.e., $\rho_1(1,h) = \rho_{01}$ or $\psi(1,h) = 0$, where $h = H/R$ is the relative height of the vessel; b) at the central point of the lower cross section of the vessel the vapor concentration differs but slightly from the equilibrium one, i.e., $\rho_1(0,0) = \rho_{1e}$ or $\psi(0, 0) = 1$. Using Eq. (11), these conditions permit us to obtain the constants in the form:

$$B = 1 - \eta_1 CA, \quad C = \frac{Ah - 1}{A(\eta_2 - \eta_1)}, \tag{18}$$

where $\eta_1 = [1/J_0(k) + 1/I_0(k)]/k^2 = -0.1579$; $\eta_2 = 2/k^2 = 0.0941$.

We will describe the convection-induced enhancement of the evaporation process by the convective Nusselt number $\mathrm{Nu} = \langle j_c\rangle/\langle j_d\rangle$, which represents the ratio of the vessel cross-sectional area-average mass fluxes of vapor during evaporation under the conditions of free convection and convective diffusion. The mean and local mass fluxes are determined by the expressions:

$$\langle j_c\rangle = \frac{1}{\pi R^2}\int_0^R j(r, z)\, 2\pi r dr, \tag{19}$$

$$j = -D\frac{\partial\rho_1}{\partial z} + \rho_1 v_z. \tag{20}$$

The latter can be written as

$$j = \frac{\rho D}{R}(\omega_{1e} - \omega_{01})\left[-\frac{\partial\psi}{\partial y} + u\left(\psi + \frac{\omega_{01}}{\omega_{1e} - \omega_{01}}\right)\right], \tag{21}$$

Having substituted this expression into Eq. (19) and performed integration with the use of [3], we obtain a formula for the mean flux of vapor $\langle j_c\rangle$ in the convective mass transfer regime:

$$\langle j_c\rangle = \frac{\rho DA}{R}(\omega_{1e} - \omega_{01})\left\{1 + \frac{C^2}{k^2}\left[\frac{J_1^2(k)}{J_0^2(k)} + \frac{I_1^2(k)}{I_0^2(k)}\right]\right\} \tag{22}$$

or

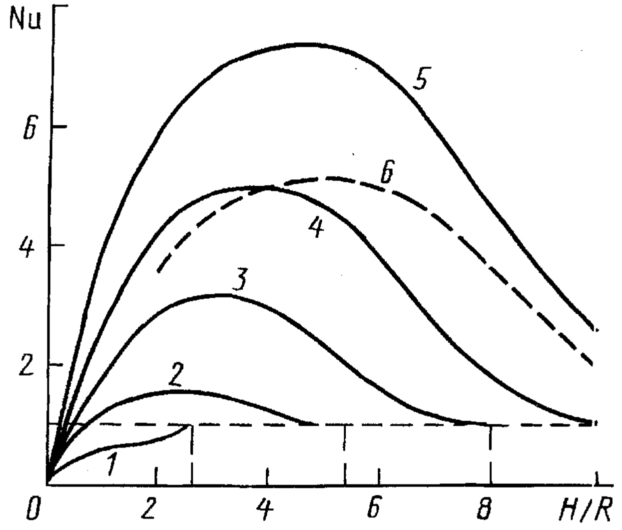
$$\langle j_c\rangle = \frac{\rho DA}{R}(\omega_{1e} - \omega_{01})\left[1 + \beta\left(h - \frac{1}{A}\right)^2\right], \tag{23}$$

where $\beta = [J_1^2(k)/J_0^2(k) + I_1^2(k)/I_0^2(k)]/[k^2(\eta_2 - \eta_1)] = 0.2914$.

A solution of the problem for the conditions of convective-diffusive transfer is known [4]; in the variables used in the present work it has the form:

$$\langle j_d\rangle = \frac{\rho D}{Rh}\ln\frac{1 - \omega_{01}}{1 - \omega_{1e}}. \tag{24}$$

Then, for the Nusselt number with account for Eq. (10) we find



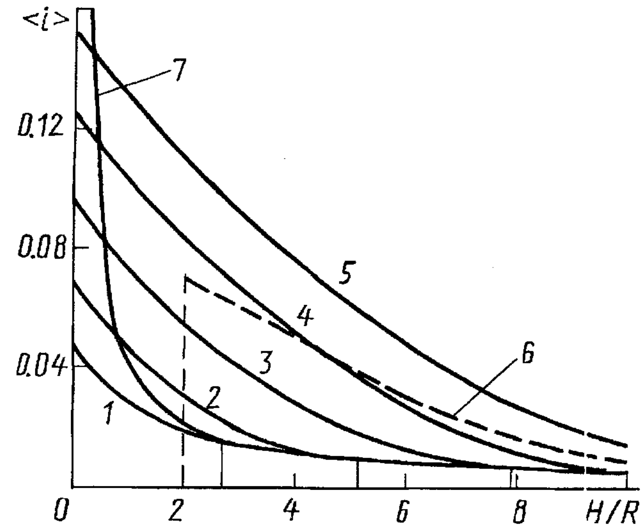


Fig. 1. Nusselt numbers as functions of the relative height of a cylindrical vessel. Values of the Grashof numbers: 1) $2 \cdot 10^3$; 2) $4 \cdot 10^3$; 3) $6 \cdot 10^3$; 4) $8 \cdot 10^3$; 5) 10^4; 6) 10^4 (according to [2]).

Fig. 2. Dimensionless mass fluxes of vapor as functions of the relative height of a cylindrical vessel. Values of the Grashof numbers: 1) $2 \cdot 10^3$; 2) $4 \cdot 10^3$; 3) $6 \cdot 10^3$; 4) $8 \cdot 10^3$; 5) 10^4; 6) 10^4 (according to [2]). Curve 7, fluxes in the case of convective-diffusive mechanism of transfer.

$$\mathrm{Nu} = \frac{\varepsilon k^4}{\mathrm{Ra}} h \left[1 + \beta \left(h - \frac{\mathrm{Ra}}{k^4} \right)^2 \right], \tag{25}$$

where $\varepsilon = (\omega_{1e} - \omega_{01}) / \ln \left[(1 - \omega_{01}) / (1 - \omega_{1e}) \right]$.

To compare with the well-known data, it is advisable to perform calculations for the water-air system considered in [2]. The authors of that work analyzed the evaporation of water at a temperature of 311 K, for which $\omega_{1e} = 0.04$, assuming the air to be dry, i.e., $\omega_{01} = 0$. For the indicated temperature the dynamic viscosity of the air is $\eta = 1.89 \cdot 10^{-5}$ Pa·sec [5]. Using the state equation for calculating the density, we obtain $\rho = 1.121$ kg/m^3 and, correspondingly, $\nu = 1.684 \cdot 10^{-5}$ m^2/sec. The coefficient of the diffusion of steam to the air is $D = 2.88 \cdot 10^{-5}$ m^2/sec [6]. This gives Pr = 0.585.

The functions Nu(h) for different values of the Grashof number given in Fig. 1 indicate that the proposed calculational technique permits one to elucidate all the main features of mass transfer during the onset of free convection in a gas-vapor mixture. The Nusselt number increases with an increase in Gr, i.e., mass transfer enhancement occurs with an increase of both the vessel radius and the difference in the mass fractions $\omega_{1e} - \omega_{01}$. In particular, for $h = 5$ at $\mathrm{Gr} = 10^4$ the free convection vaporization rate of water can increase by more than a factor of 7. However, according to the definition of the Grashof number (Eq. (10)), this increase is possible only for vessels whose radius $R > 2.26 \cdot 10^{-2}$ m. In the region of the existence of convective transfer the functions Nu (h) are extremal. The critical value of the relative height $h_{\mathrm{cr}} = \mathrm{Ra}/k^4$ (determined from the condition $u = 0$), above which the convective mixing of the gas-vapor mixture ceases, increases monotonically with an increase in Gr. When $Gr < 4 \cdot 10^3$, the gas-vapor mixture remains stable at any value of the relative height of the vessel.

The values of h at which Nu becomes smaller than unity in the left portion of Fig. 1 cannot be considered critical for the following reasons. The calculations of the fluxes $<j_c>$ and $<j_d>$ that determine the value of Nu are performed for $\psi_e = 0.04$ and $\psi_0 = 0$, just as in [2]. But while for convective transfer the condition $\psi_0 = 0$ is correct at any value of h (for a dry air), then for transfer in the convective diffusion regime this means that a zero vapor concentration must be sustained at all the points of the outlet cross section of the vessel. Obviously, its experimental realization is the more difficult, the smaller the value of h. Figure 2 illustrates the effect of h on the dimensionless mass fluxes $<i> = <j> R/\rho D$ for the convective (curves 1-5) and convective-diffusive, Eq. (7), mechanisms of transfer whose ratio also gives the value of the Nusselt number and permits one to get a more clear idea of the practical realization of the calculated functions Nu(h) in the region of small values of h. In real conditions for vessels open at the top formula (22) is inapplicable for the calculation of $<j_d>$ at small values of h, since $<j_d (h \to 0)> \to \infty$, and, consequently, there is no rapid increase in the flux $<i_d>$, as shown in Fig. 2, and it does not exceed the values of the convective flux $<i_c>$.

Figures 1 and 2 also present curves plotted from the results of numerical simulation of free convection [2] at Gr = 10^4. The satisfactory agreement of the results indicates the correctness of the analytical model considered in the present work. Its obvious advantage is the possibility to obtain simple calculating formulas permitting one to evaluate the mass transfer enhancement degree during vaporization by free concentrational convection. It should be noted that the numerical model developed in [2], does not provide the possibility of determining the critical values of the relative vessel heights h, above which no disruption of the mechanical stability of a gas-vapor mixture occurs. The proposed solution provides the possibility of such an estimation and can be used in selecting the optimal dimensions of an experimental cell for determining the diffusion coefficients of liquid vapors in gases by the Stefan method.

NOTATION

p, pressure, Pa; ρ, density, kg/m^3; v, velocity, m/sec; η, ν, dynamic and kinematic viscosity, Pa·sec, m^2/sec; D, diffusion coefficient, m^2/sec; ω_1, ω_2, mass fractions of vapor and gas in a mixture; g, free fall acceleration, m/sec^2; M_1, M_2, molar masses of vapor and gas, kg/kmole; v_r,v_z, radial and axial components of the velocity of a gas-vapor mixture, m/sec; r, z, cylindrical coordinates, m; R, H, radius and height of vessel, m; j, local mass flux of vapor, kg/(m^2·sec); $<j>$, vessel cross-sectional area-averaged mass flux of vapor, kg/(m^2·sec); $<i>$, vessel cross-sectional area-averaged mass flux.

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